

The Ultimate Solution to the Quantum Battle of the Sexes game

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Abstract. We present the unique solution to the Quantum Battle of the Sexes game. We show the best result which can be reached when the game is played according to Marinatto and Weber's scheme. The result which we put forward does not surrender the criticism of previous works on the same topic.

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1. Introduction

Theory of games concerns description of conflict situations between two or more individuals, usually called players.

Besides classical theory of games for about 10 years has been a new field of investigation - quantum games [4]. It represents an extension of traditional theory of games into the field of quantum mechanics (quantum information). In quantum games players have an access to strategies which are not encountered in the ‘macroscopic world’. This phenomenon implicates new interesting results which may be attained by players equipped with quantum strategies [2, 3, 4, 6 - 12].

1.1. Battle of the Sexes

The Battle of the Sexes is a static two-player game of nonzero sum whose matrix representation is as follows:

$$\Gamma : \begin{array}{cc} & q = 1 & q = 0 \\ \begin{array}{c} p = 1 \\ p = 0 \end{array} & \begin{bmatrix} (\alpha, \beta) & (\gamma, \gamma) \\ (\gamma, \gamma) & (\beta, \alpha) \end{bmatrix} \end{array} \quad \text{where } \alpha > \beta > \gamma.$$

Characteristic for the Battle of the Sexes game are three Nash equilibria: one is found in mixed strategies and the other two in pure strategies. The first player prefers equilibrium (1, 1) which yields him the payoff α . In turn the second player, in order to get the payoff α , prefers (0, 0). The problem of opposing expectations of the two players constitutes definite dilemma. The players, following their preferences, may play strategy profile (1, 0) that gives them the payoff γ - the least payoff in the game.

1.2. The model of quantum game

The quantum model of a two-player static game (the game in which each player chooses their strategy once and the choices of all players are made simultaneously) is a family $(\mathcal{H}, \rho_{in}, U_A, U_B, \succeq_A, \succeq_B)$ [3]. In such a model \mathcal{H} is the underlying Hilbert space of the physical system used to play a game and ρ_{in} is the initial state of this system. Sets of strategies of two players are sets U_A and U_B of unitary operators by which players can act on ρ_{in} . The symbols \succeq_A and \succeq_B mean the preference relation for the first and the second player, respectively, which can be replaced by the payoff function. The first scheme for playing quantum 2×2 game in which both players have an access to ‘quantum’ strategies appeared in [2]. In this model Hilbert space \mathcal{H} is defined as $\mathbb{C}^2 \otimes \mathbb{C}^2$. Players apply unitary operators acting on \mathbb{C}^2 which depend on two parameters. Their strategies and the initial state ρ_{in} is taken to be a maximally entangled state of two qubits. Marinatto and Weber [6] introduced a new scheme for quantizing 2×2 games. In contrary to the scheme proposed in [2], they restricted players actions to applying identity operator I and Pauli operator σ_x or any probabilistic mixture of I and σ_x . This limitation of unitary operators can lead to the situation in which the players are even unable to state whether they play a game in classical or in quantum form [11]. For this

reason Marinatto and Weber's model seems to be the more natural way for quantizing games. In the next section we give precise description of this scheme.

2. General Marinatto-Weber scheme

In the Marinatto-Weber scheme of playing 2×2 quantum games a space state of a game is the $2 \otimes 2$ dimensional complex Hilbert space with a base $(|00\rangle, |01\rangle, |10\rangle, |11\rangle)$. The initial state of a game is $|\psi_{in}\rangle = a_{00}|00\rangle + a_{01}|01\rangle + a_{10}|10\rangle + a_{11}|11\rangle$ and $I, C = \sigma_x$ are unitary operators. Players are able to manipulate the initial state $|\psi_{in}\rangle$ by employing I or C to the first and the second entry in the ket $|\cdot\rangle$, respectively. According to the idea of mixed strategies they can also apply, respectively, $pI + (1-p)C$, $qI + (1-q)C$. If the language of density matrices is used then $\rho_{in} = |\psi_{in}\rangle\langle\psi_{in}|$ and the final state of the game is as follows:

$$\begin{aligned} \rho_{fin} = & pqI_1 \otimes I_2 \rho_{in} I_1 \otimes I_2 + p(1-q)I_1 \otimes C_2 \rho_{in} I_1 \otimes C_2 \\ & + (1-p)qC_1 \otimes I_2 \rho_{in} C_1 \otimes I_2 + (1-p)(1-q)C_1 \otimes C_2 \rho_{in} C_1 \otimes C_2 \end{aligned}$$

When the original classical game is defined by a bi-matrix:

$$\Lambda : \begin{array}{cc} & \begin{array}{cc} q=1 & q=0 \end{array} \\ \begin{array}{c} p=1 \\ p=0 \end{array} & \left[\begin{array}{cc} (x_{11}, y_{11}) & (x_{12}, y_{12}) \\ (x_{21}, y_{21}) & (x_{22}, y_{22}) \end{array} \right] \end{array}$$

the payoff operators are:

$$\begin{aligned} P_A &= x_{11}|00\rangle\langle 00| + x_{12}|01\rangle\langle 01| + x_{21}|10\rangle\langle 10| + x_{22}|11\rangle\langle 11|, \\ P_B &= y_{11}|00\rangle\langle 00| + y_{12}|01\rangle\langle 01| + y_{21}|10\rangle\langle 10| + y_{22}|11\rangle\langle 11|. \end{aligned}$$

The payoff functions π_A and π_B can then be obtained as mean values of the above operators:

$$\pi_A = Tr\{P_A \rho_{fin}\}, \quad \pi_B = Tr\{P_B \rho_{fin}\}$$

After applying the procedure discussed above, the quantum equivalent of the classical game Λ is characterized by a two-dimensional bi-matrix Λ_Q , the elements of which are specified as a product of two matrices:

$$\pi(i, j) = (|a_{i \oplus_2 1, j \oplus_2 1}|^2 \quad |a_{i \oplus_2 1, j}|^2 \quad |a_{i, j \oplus_2 1}|^2 \quad |a_{i, j}|^2)(X, Y) \quad (1)$$

where: $\pi(i, j) = (\pi_A(i, j), \pi_B(i, j))$, $i, j \in \{0, 1\}$, \oplus_2 means addition modulo 2 and $(X, Y) = \left((x_{11}, y_{11}) \quad (x_{12}, y_{12}) \quad (x_{21}, y_{21}) \quad (x_{22}, y_{22}) \right)^T$.

In the special case when $|\psi_{in}\rangle = |00\rangle$ an equality $\Lambda = \Lambda_Q$ occurs.

3. Various attempts of solving the dilemma of the Quantum Battle of the Sexes game

The history of efforts put into the quantum solution to the dilemma that unavoidably occurs in the classical Battle of the Sexes began in [6], where the scheme of playing

quantum games alternative to the scheme proposed in [2] was published. Marinatto and Weber showed that the players who have an access to quantum strategies may gain the same payoff in every equilibrium. If the initial state of the game is $|\psi\rangle = (|00\rangle + |11\rangle)/\sqrt{2}$ then instead of (α, β) or (β, α) , respectively, for strategy profiles $(1, 1)$ and $(0, 0)$, they obtain $((\alpha + \beta)/2, (\alpha + \beta)/2)$. Equalization of payoffs for players obtained in both equilibria certainly eliminates differences between preferences of the players but, as Benjamin [1] correctly stated, the dilemma still exists. Despite of the fact that both players prefer two equilibrium situations to the same extent, there is still a possibility that because of lack of communication between both players they may obtain the worst payoff γ , which happens when they play combinations of strategies $(1, 0)$ or $(0, 1)$.

Further improvement in solving the dilemma of the Battle of the Sexes game was presented by Nawaz and Toor in [8]. They improved the results of [6] considering the quantum game Battle of the Sexes that begins with the initial state $|\psi_{in}\rangle = (\sqrt{5}e^{i\phi_1}|00\rangle + \sqrt{5}e^{i\phi_2}|01\rangle + e^{i\phi_3}|10\rangle + \sqrt{5}e^{i\phi_4}|11\rangle)/4$ and showing that it is equivalent to the classical game characterized by the following payoff bimatrix:

$$\Gamma_{NT} = 1/16 \begin{bmatrix} (5\alpha + 5\beta + 6\gamma, 5\alpha + 5\beta + 6\gamma) & (5\alpha + \beta + 10\gamma, \alpha + 5\beta + 10\gamma) \\ (\alpha + 5\beta + 10\gamma, 5\alpha + \beta + 10\gamma) & (5\alpha + 5\beta + 6\gamma, 5\alpha + 5\beta + 6\gamma) \end{bmatrix}$$

Then they argued that every player should choose their first strategy. It can be easily observed that [8] improve results of [6]. For any α, β, γ where $\alpha > \beta > \gamma$, it is better for both players to play ‘Nawaz and Toor’s game’ than ‘Marinatto and Weber’s game’: in [6], if players choose their strategies 1 or 0 at random, they gain with equal probability $(\alpha + \beta)/2$ and γ , what gives them the expected value $(\alpha + \beta + 2\gamma)/4$ - the result which is always worse than $(5\alpha + 5\beta + 6\gamma)/16$. However, the question arises: is Nawaz and Toor’s result the best result which players can guarantee themselves in the quantum Battle of the Sexes game?

4. Harsanyi - Selten algorithm of equilibrium selection

The algorithm of choosing equilibrium presented below is described in a renowned book by Nobel Prize winners Harsanyi and Selten [5]. Its aim is to select in each 2×2 game with two strong equilibria only one of them or the equilibrium in mixed strategies.

To demonstrate an operation of the algorithm let us consider the following 2×2 game:

$$\Delta = \begin{matrix} & q = 1 & q = 0 \\ \begin{matrix} p = 1 \\ p = 0 \end{matrix} & \begin{bmatrix} (a_{11}, b_{11}) & (a_{12}, b_{12}) \\ (a_{21}, b_{21}) & (a_{22}, b_{22}) \end{bmatrix} \end{matrix}$$

and denote by $u_1 = a_{11} - a_{21}$, $v_1 = a_{22} - a_{12}$, $u_2 = b_{11} - b_{12}$, $v_2 = b_{22} - b_{21}$. Furthermore, let us assume that the pairs of pure strategies $(1, 1)$, $(0, 0)$ form strong equilibria (analogical criterion can be formulated for equilibria placed on the second diagonal). Then there exists also the third equilibrium (s_1, s_2) in mixed strategies, where $s_1 = v_2/(u_2 + v_2)$, $s_2 = v_1/(u_1 + v_1)$.

Algorithm: From three equilibria the one which dominates according to payoffs, i.e the one, in which both players receive the largest payoffs should be chosen. If this is not a case, then the equilibrium should be chosen according to the following formula:

$$(r_1, r_2) = \begin{cases} (1, 1), & \text{if } u_1 u_2 > v_1 v_2 \\ (0, 0), & \text{if } u_1 u_2 < v_1 v_2 \\ (s_1, s_2), & \text{if } u_1 u_2 = v_1 v_2. \end{cases}$$

Such strategy pair is called as risk-dominant equilibrium [5].

It is important to notice that the given algorithm is not contradictory to individual rationality. The algorithm should not be treated as an oracle which gives players unjustified hints which are in conflict with common sense. The criterion entirely reflects rational behavior of the players (see comments in [5]).

In order to see how this algorithm works, we apply it to the quantum version of the game the Battle of the Sexes studied by Nawaz and Toor in [8] and described by the payoff bi-matrix Γ_{NT} .

It can be easily noticed that the game Γ_{NT} has three equilibria but none of them is dominant according to payoffs. Since $u_1 = u_2 = 4(\alpha - \gamma)$ and $v_1 = v_2 = 4(\beta - \gamma)$, we get $u_1 u_2 = 16(\alpha - \gamma)^2 > 16(\beta - \gamma)^2 = v_1 v_2$. Therefore according to the rule given by the Harsanyi and Selten's algorithm, players should chose the equilibrium $(1, 1)$ - a strategy pair which also Nawaz and Toor consider as the only rational solution in this game.

5. Dilemma of the Battle of the Sexes overcome

In the previous section we presented the algorithm of equilibrium selection which should be adapted by rational players for 2×2 games with two strong equilibria. The quantum game begins when players receive initial state and at this stage there is a need to define precisely its shape. Below-given lemma will allow one to use the Harsanyi-Selten algorithm and also to equalize players' preferences.

Let an initial state $|\psi_{in}\rangle = a_{00}|00\rangle + a_{01}|01\rangle + a_{10}|10\rangle + a_{11}|11\rangle$ of a quantum 2×2 game played according to the Marinatto-Weber scheme be given. Then the original classical game Γ transforms into the game Γ' such that the following lemma holds:

Lemma 5.1 *If $|a_{00}|^2 = |a_{11}|^2 = \frac{1}{2}(1 - (\epsilon_1 + \epsilon_2))$, $|a_{01}|^2 = \epsilon_1$, $|a_{10}|^2 = \epsilon_2$, where $\epsilon_1 + \epsilon_2 \leq 1 - 2 \max\{\epsilon_1, \epsilon_2\}$ for $\epsilon_1 \neq \epsilon_2$ and $\epsilon < 1/4$ for $\epsilon_1 = \epsilon_2 = \epsilon$ then for any real numbers $\alpha > \beta > \gamma$*

- a) *a game Γ' is identical to Γ with respect to strategy profiles which constitute Nash equilibria in pure strategies and with respect to the number of equilibria,*
- b) *payoff functions π'_A , π'_B of the quantum game Γ' fulfill the condition:
 $\pi'_A(r_1, r_2) = \pi'_B(r_1, r_2)$ for all equilibria (r_1, r_2) of the game Γ' .*

Proof: Insert $|a_{00}|^2 = |a_{11}|^2 = \frac{1}{2}(1 - (\epsilon_1 + \epsilon_2))$, $|a_{01}|^2 = \epsilon_1$, $|a_{10}|^2 = \epsilon_2$ to the formula (1). Taking into account assumptions of the lemma about the sum $\epsilon_1 + \epsilon_2$ we obtain:

$$\begin{aligned} \pi'_A(1, 1) - \pi'_A(0, 1) &= \pi'_B(1, 1) - \pi'_B(1, 0) = \\ &(\alpha - \gamma)\left(\frac{1}{2}(1 - (\epsilon_1 + \epsilon_2)) - \epsilon_2\right) + (\beta - \gamma)\left(\frac{1}{2}(1 - (\epsilon_1 + \epsilon_2)) - \epsilon_1\right) > 0. \end{aligned} \quad (2)$$

Similarly:

$$\begin{aligned} \pi'_A(0, 0) - \pi'_A(1, 0) &= \pi'_B(0, 0) - \pi'_B(0, 1) = \\ &(\alpha - \gamma)\left(\frac{1}{2}(1 - (\epsilon_1 + \epsilon_2)) - \epsilon_1\right) + (\beta - \gamma)\left(\frac{1}{2}(1 - (\epsilon_1 + \epsilon_2)) - \epsilon_2\right) > 0. \end{aligned} \quad (3)$$

We infer from this results that pairs $(1, 1)$, $(0, 0)$ form Nash equilibria and none of the strategies is weakly dominated. Therefore, the game Γ' possesses also an equilibrium in mixed strategies.

Furthermore, it can be easily observed that $\pi'_A(1, 1) = \pi'_B(1, 1) = \pi'_A(0, 0) = \pi'_B(0, 0)$. Let us mark by (s_1, s_2) the third equilibrium of the game Γ' . Due to $u_1 = u_2$ and $v_1 = v_2$, we obtain equality $(s_1, s_2) = (s_1, s_1) = (s_2, s_2)$. Therefore, besides the equalities: $\pi'_A(1, 0) = \pi'_B(0, 1)$ and $\pi'_A(0, 1) = \pi'_B(1, 0)$ we get $\pi'_A(s_1, s_2) = \pi'_B(s_1, s_2)$.

The essential assumptions of the lemma are not *condito sine qua non* to fulfill the thesis. Taking into consideration, for example, another initial state: $|\psi_{in}\rangle = a_{01}|00\rangle + a_{00}|01\rangle + a_{11}|10\rangle + a_{10}|11\rangle$ one obtains a game which is identical to Γ' up to relabelling of strategies of one of the players. Moreover, the assumption $\epsilon_1 + \epsilon_2 \leq 1 - 2 \max\{\epsilon_1, \epsilon_2\}$ can be weakened. The assumptions define form of the initial state for the quantum Battle of the Sexes game with any $\alpha > \beta > \gamma$. The necessary and sufficient condition for inequalities (2) and (3) to be true require dependence of ϵ_1 and ϵ_2 on α , β and γ . However for simplifying the results, we will not go into details of this problem. As we will notice further the most important for our study are only values of (ϵ_1, ϵ_2) in the neighborhood of $(0, 0)$.

One of the characteristic features of both the classical game the Battle of the Sexes and any of its quantum versions is the lack of any equilibria which are dominating according to payoffs. However, the following theorem states that when assumptions of the lemma are fulfilled, in quantum version of this game an risk-dominant equilibrium exists.

Theorem 5.2 *If the quantum version Γ' of the game Γ fulfills assumptions of the lemma, then its risk-dominant equilibrium is the strategy profile:*

$$(r_1, r_2) = \begin{cases} (1, 1), & \text{when } \epsilon_1 > \epsilon_2 \\ (0, 0), & \text{when } \epsilon_1 < \epsilon_2 \\ (1/2, 1/2), & \text{when } \epsilon_1 = \epsilon_2. \end{cases} \quad (4)$$

Proof: Let us calculate $u_1 u_2$, and $v_1 v_2$ from the algorithm and estimate the difference $u_1 u_2 - v_1 v_2$:

$$u_1 u_2 = \left[(\alpha - \gamma)\left(\frac{1}{2}(1 - (\epsilon_1 + \epsilon_2)) - \epsilon_2\right) + (\beta - \gamma)\left(\frac{1}{2}(1 - (\epsilon_1 + \epsilon_2)) - \epsilon_1\right) \right]^2,$$

$$v_1 v_2 = \left[(\alpha - \gamma) \left(\frac{1}{2} (1 - (\epsilon_1 + \epsilon_2)) - \epsilon_1 \right) + (\beta - \gamma) \left(\frac{1}{2} (1 - (\epsilon_1 + \epsilon_2)) - \epsilon_2 \right) \right]^2,$$

consequently:

$$u_1 u_2 - v_1 v_2 = (\alpha + \beta - 2\gamma)(\alpha - \beta)(1 - 2(\epsilon_1 + \epsilon_2))(\epsilon_1 - \epsilon_2).$$

The first and the second element of the product is surely positive. Due to the assumption of the lemma $1 - 2(\epsilon_1 + \epsilon_2)$ is also positive. Therefore, the sign of the difference $u_1 u_2 - v_1 v_2$ depends only on the sign of the difference $\epsilon_1 - \epsilon_2$.

In the case when $\epsilon_1 = \epsilon_2$ the game Γ' is characterized by the following equalities:

$$\pi'_A(r_1, r_2) = \pi'_B(r_1, r_2) \quad \text{for all } (r_1, r_2)$$

$$\pi'(i, j) = \pi'(i \oplus 1, j \oplus 1) \quad \text{for all } i, j \in \{0, 1\}$$

which implicate that Nash equilibrium in mixed strategies is formed by a pair of strategies $(1/2, 1/2)$.

The initial state is known to the players, so according to the theorem it determines all the development of the game. The values of payoff function corresponding to (4) are as follows:

$$\pi'_{A,B}(r_1, r_2) = \begin{cases} \frac{1}{2} [(\alpha + \beta) - (\alpha + \beta - 2\gamma)(\epsilon_1 + \epsilon_2)], & \text{when } \epsilon_1 > \epsilon_2 \\ \frac{1}{2} [(\alpha + \beta) - (\alpha + \beta - 2\gamma)(\epsilon_1 + \epsilon_2)], & \text{when } \epsilon_1 < \epsilon_2 \\ \frac{1}{4}(\alpha + \beta + 2\gamma) - \frac{1}{2}\epsilon\gamma, & \text{when } \epsilon_1 = \epsilon_2 = \epsilon. \end{cases}$$

Payoff function depends only on the values of ϵ_1, ϵ_2 , thus it can be identified with a function of two variables ϵ_1 and ϵ_2 :

$$\pi'_{A,B}(\epsilon_1, \epsilon_2) = \begin{cases} \frac{1}{2} [(\alpha + \beta) - (\alpha + \beta - 2\gamma)(\epsilon_1 + \epsilon_2)], & \text{when } \epsilon_1 \neq \epsilon_2 \\ \frac{1}{4}(\alpha + \beta + 2\gamma) - \frac{1}{2}\epsilon\gamma, & \text{when } \epsilon_1 = \epsilon_2 = \epsilon. \end{cases}$$

This function is composed of two linear functions with variables ϵ_1, ϵ_2 . Let us examine its limit:

$$\lim_{(\epsilon_1, \epsilon_2) \rightarrow (0,0)^+} \pi'_{A,B}(\epsilon_1, \epsilon_2) = \begin{cases} \frac{1}{2}(\alpha + \beta), & \text{when } \epsilon_1 \neq \epsilon_2 \\ \frac{1}{4}(\alpha + \beta + 2\gamma), & \text{when } \epsilon_1 = \epsilon_2 = \epsilon. \end{cases}$$

It follows, that:

$$\sup_{\epsilon_1, \epsilon_2} \pi'_{A,B}(\epsilon_1, \epsilon_2) = \frac{1}{2}(\alpha + \beta).$$

The maximum value of the function $\pi'_{A,B}(\epsilon_1, \epsilon_2)$ does not exist, but for any small positive value δ an arbiter is able to prepare the initial state with sufficiently small ϵ_1, ϵ_2 that are different from each other in such way that payoffs of players differ from $\frac{1}{2}(\alpha + \beta)$ less than δ . This means that in the quantum Battle of the Sexes game both players may obtain equal payoffs arbitrary close to $\frac{1}{2}(\alpha + \beta)$.

Example 5.3 If $(\alpha, \beta, \gamma) = (5, 3, 1)$, then according to the result of Nawaz and Toor each player gets payoff 2,875 while our formula yields for an initial state of the game characterized by $|a_{01}|^2 = \epsilon_1 = 0,01$, $|a_{10}|^2 = \epsilon_2 = 0,02$ and $|a_{00}|^2 = |a_{11}|^2 = 0,485$ payoffs $[(5 + 3) - 0,03(5 + 3 - 2)]/2 = 3,91$.

6. Conclusion

We obtained a new result in the quantum Battle of the Sexes game played according to Marinatto-Weber scheme. In contrast to [6] we considered the initial state of the game to be most general state of two qubits. We put conditions for amplitudes of the initial state so that quantum form of the Battle of the Sexes game has identical strategic positions of players as the initial game. Differently from [8], we did not select a particular initial state, but we examined the dependence of players' payoffs on amplitudes of base states that form the initial state of the game. Our research showed that the initial state $|\psi_{in}\rangle = a_{00}|00\rangle + a_{01}|01\rangle + a_{10}|10\rangle + a_{11}|11\rangle$ from [8] characterized by $|a_{00}|^2 = |a_{11}|^2 = |a_{01}|^2 = 5/16$, $|a_{10}|^2 = 1/16$ is one of many initial states, which can be prepare without lost characteristic feature of the classical Battle of the Sexes game. Moreover, we discovered infinitely more initial states for which players can achive higher payoffs than by means of Nawaz and Toor's initial state. This allowed to determine the supremum of the payoffs values. This quantum version assure that its participants can get payoffs arbitrary close to the maximal payoff possible in the game: $\frac{1}{2}(\alpha + \beta)$, which is the highest value that can be obtained in the 'classical' game if and only if players are allowed to communicate.

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